6.1 Notes: Vectors in a Plane

Quantities such as force and velocity involve both ________________ and ________________ and cannot be completely characterized by a single real number. To represent such a quantity, you can use a ________________ ___________ ________________. Segment $\overrightarrow{PQ}$ has ________________ ___________ P and ________________ ___________ Q. Its ________________ (or length) is denoted by ___________ and can be found using the Distance Formula.

Two directed line segments that have the same magnitude and direction are ________________. The set of all directed line segments that are equivalent to the directed line segment t $\overrightarrow{PQ}$ is a ________________ ______ ______ ________________, written **v** = $\overrightarrow{PQ}$. Vectors are denoted by ________________, ________________ letters such as **u, v,** and **w.**

Ex: 1 Show that **u** and **v** in figure 6.13 are equivalent. (Show both magnitude and direction)

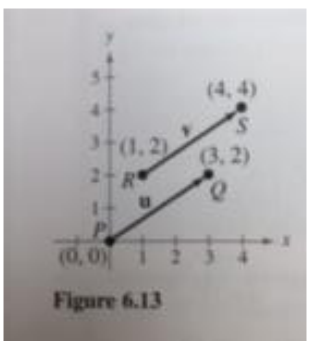


Figure 6.13

Component Form of a Vector

The directed line segment whose initial point is the ________________ is often the most convenient representative of a set of equivalent directed line segments. This representative of the vector **v** is in ________________ ________________.

A vector whose initial point is the origin (0, 0) can be uniquely represented by the coordinates of its terminal point (v_1, v_2). This is the ________________ ___________ **of a vector v,** written as **v** = $\langle \boldsymbol{v_1}, \boldsymbol{v_2} \rangle$**.** The coordinates v_1 and $v_2$are the ________________ of **v.** If both the initial point and the terminal point lie at the origin, then **v** is the ________________ ________________ and is denoted by **0** = $\langle \mathbf{0}, \mathbf{0} \rangle$**.**

Component Form of a Vector

The component form of the vector with initial point P(p_1, p_2) and terminal point Q(q_1, q_2) is given by

$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \boldsymbol{v}$$

The **magnitude** (or length) of **v** is given by

$$\|\boldsymbol{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}\,.$$

If $\|\boldsymbol{v}\| = 1$, then **v** is a **unit vector.** Moreover $\|\boldsymbol{v}\| = 0$ if and only if **v** is the zero vector **0.**

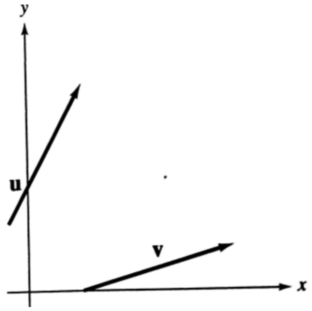
Ex: 2 Find the component form and magnitude of the vector **v** that has the initial point (-3, 2) and terminal point (8, 2).

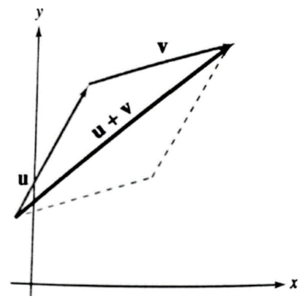
You Try: Find the component form and magnitude of the vector **v** that has the initial point (-2, 3) and terminal point (-7, 9).

Vector Operations

The two basic vector operations are ________________ ________________ and ________________ ________________.In operations with vectors, numbers are usually referred to as ________________. In this text scalars will always be real numbers. Geometrically, the product of vector **v** and scalar k is the vector that is $|k|$times as long as **v**. When k is ________________, k**v** has the same direction as **v**, and when k is ________________ k**v** has the direction opposite that of **v**. See figure 6.14 in your textbook.

To add two vectors **u** and **v** geometrically, first position them(without changing their lengths or directions) so that the initial point of the second vector **v** coincides with the terminal point of the first vector **u**. The sum **u + v** is the vector formed by joining the initial point of the first vector **u** with the terminal point of the second vector **v**.





This technique is called the ________________ __________ for vector addition because the vector **u + v**, is often called the ________________ of vector addition, is the diagonal of a parallelogram having adjacent sides **u** and **v**.

Definitions of Vector Addition and Scalar Multiplication

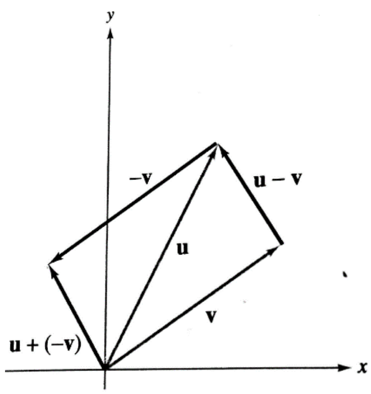
Let **u** = $\langle \boldsymbol{u_1}, \boldsymbol{u_2} \rangle$ and **v** = $\langle \boldsymbol{v_1}, \boldsymbol{v_2} \rangle$ be vectors and let *k* be a scalar (a real number).

Then the **sum** of **u** and **v** is the vector **u + v** = $\langle \boldsymbol{u_1} + \boldsymbol{v_1}, \boldsymbol{u_2} + \boldsymbol{v_2} \rangle$ and

the **scalar multiple** of *k* times **u** is the vector *k***u** = $k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle$.

The **negative** of **v** = $\langle \boldsymbol{v_1}, \boldsymbol{v_2} \rangle$ is $-\boldsymbol{v} = (-1)\boldsymbol{v} = \langle -v_1, -v_2 \rangle$ and

The **difference** of **u** and **v** is $\boldsymbol{u} - \boldsymbol{v} = \boldsymbol{u} + (-\boldsymbol{v}) = \langle u_1 - \boldsymbol{v}_1, u_2 - v_2 \rangle$**.**



To represent $\boldsymbol{u} - \boldsymbol{v}$ geometrically, you can use directed line segments with the **same** initial point. The difference $\boldsymbol{u} - \boldsymbol{v}$ is the vector from the terminal point of **v** to the terminal point of **u**, which is equal to $\boldsymbol{u} + (-\boldsymbol{v})$**.**

Ex: 3 Let **v** = $\langle \boldsymbol{-2}, \boldsymbol{5} \rangle$ and **w** = $\langle \boldsymbol{3}, \boldsymbol{4} \rangle$**.** Find each of the following vectors.

a. 2**v** b. **w** – **v** c. **v** + 2**w**

You Try: Let **u** = $\langle \boldsymbol{1}, \boldsymbol{4} \rangle$ and **v** = $\langle \boldsymbol{3}, \boldsymbol{2} \rangle$**.** Find each of the following vectors.

a. **u** + **v** b. **u** – **v** c. 2**u** – 3**v**

Properties of vector Addition and Scalar Multiplication		
Let **u, v,** and **w** be vectors and let *c* and *d* be scalars. Then the following properties are true.		
1. **u + v = v + u**	**2. (u + v) + w = u + (v + w)**	**3. u + 0 = u**
4. u + (-u) = 0	**5.** $c(d\mathbf{u}) = (cd)\mathbf{u}$	**6.** $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$	**8.** $1(\mathbf{u}) = \mathbf{u}$, $0(\mathbf{u}) = \mathbf{0}$	**9.** $\|c\boldsymbol{v}\| = |c|\|\boldsymbol{v}\|$

Unit Vectors

In many applications, it is useful to find a unit vector that has the same direction as a given nonzero vector **v**. To do this, you can divide **v** by its magnitude to obtain

$$\boldsymbol{u} = unit\ vector = \frac{v}{\|\boldsymbol{v}\|} = \left(\frac{1}{\|v\|}\right)\boldsymbol{v}$$

Note that **u** is a scalar multiple of **v**. The vector **u** has a magnitude of **1** and the same direction as **v**. The vector **u** is called a ______________ ______________ _____ _____ ________________ _____ _____.

Ex: 4 Find a unit vector in the direction of **v** = $\langle \mathbf{7}, -\mathbf{3} \rangle$ and verify that the result has a magnitude 1.

You Try: Find a unit vector in the direction of **v** = $\langle \mathbf{6}, -\mathbf{1} \rangle$ and verify that the result has a magnitude 1.

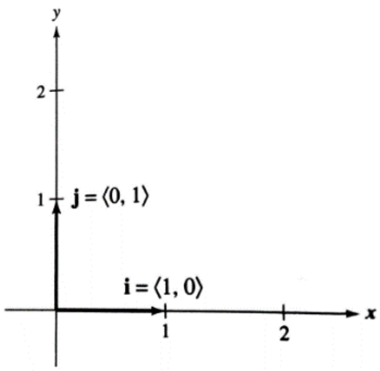


Figure 6.19

The unit vectors $\langle 1, 0\rangle$ and $\langle 0, 1\rangle$ are called _______________ __________ ___________ and are denoted by **i** = $\langle 1, 0\rangle$ and **j** = $\langle 0, 1\rangle$ as shown in Figure 6.19.

These vectors can be used to represent any vector $\vec{v} = \langle v_1, v_2\rangle$.

$$\vec{v} = \langle v_1, v_2\rangle = v_1\langle 1,0\rangle + v_2\langle 0,1\rangle = v_1\vec{\boldsymbol{\imath}} + v_2\vec{\boldsymbol{\jmath}}$$

The scalars v_1 and v_2 are called the **horizontal** and **vertical** components of $\vec{\boldsymbol{v}}$.

$v_1\vec{\boldsymbol{\imath}} + v_2\vec{\boldsymbol{\jmath}}$ is called a **linear combination** of the vectors $\vec{\boldsymbol{\imath}}$ and $\vec{\boldsymbol{\jmath}}$.

Any vector in the plane can be written as a linear combination of the standard unit vectors $\vec{\boldsymbol{\imath}}$ and $\vec{\boldsymbol{\jmath}}$.

Ex: 5 Let **u** be a vector with initial point (5, 2) and terminal point (-3, -1). Write **u** as a linear combination of the standard unit vectors **i** and **j.**

You Try: Let **u** be a vector with initial point (-2, 6) and terminal point (-8, 3). Write **u** as a linear combination of the standard unit vectors **i** and **j.**

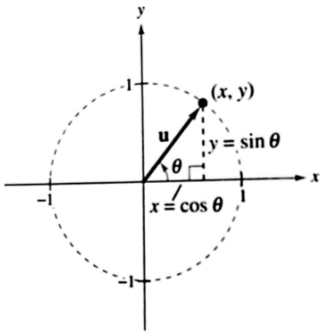
Ex: 6 Let **u =** -3**i** + 8**j** and **v** = 2**i** – **j.** Find 2**u** – 3**v.**

You Try: Let **u = i** – 2**j** and **v** = -3**i** + 2**j.** Find 5**u** – 2**v.**

Direction Angles

If **u** is a unit vector such that θ is the angle (measured counterclockwise) from the positive x-axis to **u**, then the terminal point of **u** lies on the unit circle and you have

$\mathbf{u} = \langle x, y\rangle = \langle \cos\theta, sin\theta\rangle = (cos\theta)\boldsymbol{i} + (sin\theta)\boldsymbol{j}$ see Figure 6.20



$\|\mathbf{u}\| = 1$

Figure 6.20

The angle θ is the _______________ _______________ of the vector **u**.

Suppose that **u** is a unit vector with direction angle θ. If **v** = a**i** + b**j** is any vector that makes an angle θ with the positive x-axis, then it has the same direction as **u** and **v** = $\|v\|\langle cos\theta, sin\theta\rangle = \|v\|(cos\theta)\boldsymbol{i} + \|v\|(sin\theta)\boldsymbol{j}$.

Since **v** = a**i** + b**j** = $\|v\|(cos\theta)\boldsymbol{i} + \|v\|(sin\theta)\boldsymbol{j}$, then the direction angle θ for **v** is determined from

$$tan\theta = \frac{sin\theta}{cos\theta} = \frac{\|\boldsymbol{v}\|sin\theta}{\|\boldsymbol{v}\|cos\theta} = \frac{b}{a}$$

Ex: 7 Find the direction angle of each vector. a. $\boldsymbol{u} = 3\boldsymbol{i} + 3\boldsymbol{j}$ b. $\boldsymbol{v} = 3\boldsymbol{i} - 4\boldsymbol{j}$

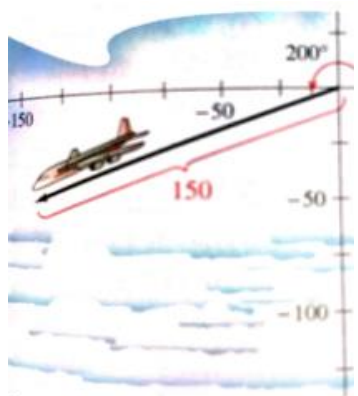
You Try: Find the direction angle of each vector. a. $\boldsymbol{v} = -6\boldsymbol{i} + 6\boldsymbol{j}$ b. $\boldsymbol{v} = -7\boldsymbol{i} - 4\boldsymbol{j}$

The **velocity** of a moving object is a vector because velocity has both magnitude and direction. The magnitude of velocity is **speed**.

Bearing is the angle that the line of travel makes with due North, measured clockwise. Bearing and the direction angle are not the same angle!

Ex. 8 A DC-10 jet aircraft is flying on a bearing of $305°$ at 520 mph. Find the component form of the velocity of the airplane.

Ex: 9 Find the component form of the vector that represents the velocity of an airplane descending at a speed of 150 miles per hour at an angle $20°$ below the horizontal.



igure 6.23

Ex: 10 An airplane is traveling at a speed of 500 miles per hour with a bearing of 330°. During take-off, the plane encounters a 70 mph wind in the direction N 45° E. Find the ground speed and direction of the airplane.

1st) Draw a diagram.

2nd) Find the component form velocity of the airplane without the effect of wind. (a)

3rd) Find the component form velocity of the wind alone (b)

4th) The true velocity of the airplane (with the effect of wind), in component form is v = a + b

5th) The <u>ground speed</u> is the magnitude of v.

6th) The <u>direction</u> of the airplane should be given as a bearing. So, find the direction angle, then convert the direction angle to a bearing.